Mobile Termination and Consumer Expectations under the Receiver-Pays Regime

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February 2011

Abstract

We analyze how termination charges affect retail prices when taking into account that receivers derive some utility from a call and when firms may charge consumers for receiving calls. We assume that consumers form expectations about network sizes in a passive, but ex-post rational way. We show that RPP enlarges the set of equilibria compared to CPP. For a given termination charge and inelastic subscription demand, RPP allows firms to obtain higher profits at the expense of consumers. Socially optimal termination charges are below cost and lower under CPP than under RPP. We also analyze elastic subscription demand. Total surplus, consumer surplus, and market penetration are all maximized by the same positive but below cost termination charge. Firms’ profits typically increase when the termination charged is removed away from the socially optimal one, in either direction.

Keywords: Bill and Keep; Call externality; Access Pricing; Interconnection; Receiver pays; Consumer Expectations

JEL classification: D43; K23; L51; L96

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1 Introduction

Although the telecommunications sector has been liberalized in most industrialized countries, some regulation remains. A clear example is call termination on mobile telephone networks. Mobile operators must interconnect their networks so that their customers can communicate with the customers of other networks. This requires mobile operators to provide a wholesale service called ‘call termination’, whereby each completes a call made to one of its subscribers by a caller on another network. Call termination is provided in exchange for a fee.\footnote{The fee is sometimes equal to zero. This arrangement is known as Bill and Keep.} This fee, also called mobile termination rate, is paid by the originating operator to the terminating operator. Since the market for termination is monopolistic, one cannot rely on competition to get termination rates at the efficient level. Excessive termination rates are believed to inflate retail prices so that usage is inefficiently low. The European Commission has urged national regulators to step in and regulate these termination rates towards the true cost. Most countries around the world have followed suit and do regulate termination rates. Regulators often use so called gliding paths which reduce termination rates gradually over a period of several years. At present termination rates in most countries are still believed to be above the cost of termination and regulators intend to reduce them further over the next years.

In May 2009, the European Commission recommended national regulatory authorities to set termination rates based on the costs (i.e., the actual incremental cost of providing call termination – without allowing for common costs) incurred by an efficient operator.\footnote{Commission Recommendation of 7 May 2009 on the Regulatory Treatment of Fixed and Mobile Termination Rates in the EU (2009/396/EC).} The European Commission’s view was also supported by the European regulators group, who in the Common Position adopted on February 2008\footnote{See “ERG’s Common Position on symmetry of fixed call termination rates and symmetry of mobile call termination rates”, adopted by the ERG-Plenary on 28th February 2008, p. 4-5. Available at http://www.erg.eu.int.} decided to take a position in favor of setting a unique and uniform termination rate for all network operators at the cost incurred by an hypothetical efficient operator. As a result, the average MTR in Europe could drop from about 8.55 euro cents per minute at the end of 2009, to approximately 2.5 euro cents per minute by 2012 [see Harbord and Pagnozzi, 2010]. In light of these announcements, Vodafone and other large European mobile operators warned the European Commission that cutting termination rates could mean the end of handset subsidies for consumers and lead to a price increase. Furthermore, Vodafone claimed that cutting termination rates could result in a US style business model, where users pay for both placing and receiving calls.

The burden of regulation of termination rates is quite high. Any attempt by a national regulator to lower termination charges has to be preceded by a formal investigation of the
relevant market and a round of public consultations. Operators oppose cuts in termination rates and challenge any argument made by regulators. Often there are disputes about what the true costs are, how they should be calculated and also about what the real effect of lower termination charges is. In countries as the US and Canada, however, there seems to be no need for regulators to set termination rates. In these countries the operators must negotiate reciprocal termination rates between themselves and voluntarily agree to set them very low, sometimes even at zero (that is, a Bill and Keep regime is chosen). Another major difference between these countries and the European markets is that a so called Receiving Party Pays (RPP) regime is used, while in Europe a Calling Party Pays (CPP) regime is in place. RPP means that operators charge a price to their customers not only for placing calls but also one for receiving calls. In this way operators can recover the cost of termination from their own customers. Littlechild (2006) argues that RPP countries have lower usage prices and higher usage than CPP countries, but higher fixed fees (or lower hand-set subsidies) and perhaps lower penetration rates or at least slower growth in penetration rates. Dewenter and Kruse (2010) argue that penetration rates in CPP and RPP countries are not significantly different once one controls for endogenous regulation.

The seeming superiority of the RPP regime has lead some economists to call upon regulators in CPP countries to impose an RPP regime. However, the statistical evidence of correlations between the payment regimes, termination charges, penetration, and retail prices does not imply there is a specific causal relationship. In particular, it is not clear that using an RPP regime will bring all the benefits that seem to be correlated with RPP regimes. In fact, the CPP regime can be considered as a special case of RPP where consumers happen to be charged a zero price for reception. Moreover, nothing prevents operators in RPP countries without regulated termination rates to agree upon high termination charges. And even if RPP were superior from a social welfare point of view, it would be difficult to imagine that regulators could actually force firms to use such specific pricing structure. Only if it is in the firms’ interest in terms of profitability, RPP regimes will be used. In our view, regulators can at most influence the choice of firms between CPP and RPP by setting adequate termination rates. However, inducing RPP regimes should not be the objective of regulators per se. Regulators should set termination rates such that the resulting outcome in terms of retail prices is socially efficient.

The impact of termination rates on competition, profit and welfare has been extensively studied. This burgeoning literature starts with the seminal works of Armstrong (1998) and Laffont, Rey and Tirole (1998a, b) (henceforth ALRT). Most of the theoretical work on

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4See for example. De Bijl et al. (2005).

5For a complete review of the literature on access charges see Armstrong (2002), Vogelsang (2003) and
mobile network interconnection typically assumes that consumers derive utility only from making calls, ignoring the existence of call externalities — that is, the fact that not only callers but also receivers of a call enjoy a positive benefit. Clearly, if there is no utility at all for receiving calls, consumers would refuse to answer the phone if they have to pay a positive price for it, so that only the CPP regime makes sense. Still, the results obtained in the literature assuming CPP may be relevant also when call externalities exist and RPP is allowed since it could be the case that networks find it optimal to charge a zero price for reception. For example, Laffont et al. (1998b) consider the case when networks compete in nonlinear prices and can charge different price for on- and off-net calls. They show that profit is strictly decreasing in termination charge. Building on their analysis, Gans and King (2001) show that firms using a CPP regime strictly prefer below cost termination charges. The intuition for their results is that if termination charge is above cost, off-net calls will be more expensive than on-net calls. As there is a price differential between on- and off-net calls, consumers care about the size of each network (the so-called ‘tariff-mediated network externalities’). In particular, they will be more eager to join the larger network. Consequently, acquisition costs are reduced, which in turn intensifies competition for subscribers and results in lower subscription fees and profits. Firms would thus prefer termination charges below cost. As total welfare would be maximized by termination charges equal to cost, this implies that consumers are better off when termination charges are strictly above cost. Berger (2005) considers the same setting where firms again use CPP but where call externalities do exist. In this case the social welfare maximizing termination charge is shown to be below cost. Berger (2005) argues that regulation is not necessary as the preferences of firms and regulators are aligned. As a matter of fact, these results are at odds with real world observations since regulators around the world, and especially in the European Union, are concerned about too high termination charges and operators consistently oppose cutting termination rates.  

The possibility that the receiving party enjoys benefits from a call is clearly important for the manner in which firms compete in the retail market. Once it is recognized that consumers

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6One assumption that is invoked to justify the absence of call externalities in models of network competition is that call externalities could be largely internalized by the parties (see Competition Commission [2003, paras 8.257 to 8.260]). However, as argued by Hermalin and Katz (2004, p. 424), “this assumption is applicable only to a limited set of situations in which either the communicating parties behave altruistically or have a repeated relationship”. Additionally, Harbord and Pagnozzi [2010] argue that the empirical basis for the internalization of call externalities is unclear.

7Nevertheless, this result has been shown to be very robust. For example, it holds for any number of networks [Calzada and Valletti, 2008] and when networks are asymmetric [López and Rey, 2009]. Also, Hurkens and Jeon (2009) show that this result holds when there are both network externalities (i.e., elastic subscription demand as in Dessein, 2003) and network-based price discrimination.
enjoy benefits from receiving a call, it follows that they are prepared to pay for this. Indeed, in some countries (e.g. Canada, Singapore, Hong Kong and the United States) mobile operators charge their subscribers for the calls they receive.\textsuperscript{8} An incipient literature has started to examine the relationship between termination rates and equilibrium prices in an environment with call externalities and RPP regimes. Laffont et al. (2003; LMRT hereafter), Jeon et al. (2004; JLT hereafter), Hermalin and Katz (2006), Cambini and Valletti (2008) and López (2010) are the papers closest to ours.\textsuperscript{9} LMRT analyze Internet backbone competition and assume that there exist two types of users: websites (senders) and consumers (receivers). Hermalin and Katz study whether termination charges can induce carriers to internalize the externalities that arise when both senders and receivers of telecommunications messages enjoy benefits. But in contrast to the framework of LMRT, in which there are two different types of users, they consider that any given user has a one-half chance of being a sender and a one-half chance of being a receiver. In JLT, López (2010)\textsuperscript{10}, and this paper, however, every consumer both sends and receives traffic, and moreover obtains surplus from and is charged for placing and receiving calls. JLT and López (2010) consider duopoly markets and observe that multiple equilibria exist. They introduce noise in the utility functions in order to select a unique one. Based upon this selection criterion, they obtain the following results. On the one hand, in the absence of network-based price discrimination, mobile operators charge calls and call receptions at their off-net cost.\textsuperscript{11} Hence operators charge a positive price for incoming calls only when the termination charge is below cost (so as to recover the cost of providing the service of call termination).\textsuperscript{12} When termination charge is above cost operators will set negative prices and subsidize incoming calls in order to earn termination profits. On the other hand, when mobile operators can differentiate their calling and reception charges according to whether the communication is on- or off-net, connectivity is prone to breakdown. The reason is that off-net calling and reception charges allow network operators to create direct externalities on the customers of rival operators.

In the present paper we develop further and generalize the analysis of JLT and obtain new

\textsuperscript{8}According to Dewenter and Kruse (2010) 14 countries used RPP from the beginning at least until 2003. Another 31 countries started with RPP but switched at some point to CPP.

\textsuperscript{9}Other related papers in this literature include Kim and Lim (2001); DeGraba (2003); Hahn (2003); Berger (2004, 2005); Hermalin and Katz (2001, 2004)).

\textsuperscript{10}López (2010) generalizes the framework of JLT by allowing a random noise in both the callers’ and receivers’ utilities, by removing (at some stages) the assumption of a given proportionality between the utility functions, and by allowing asymmetry between mobile operators with respect to the number of locked-in customers.

\textsuperscript{11}This so called “off-net-cost pricing principle” dates back to LMRT, who found this pricing rule in a framework for Internet backbone competition.

\textsuperscript{12}Cambini and Valletti (2008) obtain the same result in their framework of information exchange between calling parties with interdependency among outgoing and incoming calls.
results that have implications for retail pricing. First, rather than a duopolistic Hotelling model we consider a more general Logit model that allows for any number of firms and also for the possibility of elastic subscription demand, that is where consumers may choose to stay unsubscribed. A further novelty of our analysis lies in studying how consumer expectations affect equilibrium end-user prices (and so equilibrium profit and welfare). We introduce this novelty because we have shown elsewhere that consumer expectations are crucial under CPP regimes. A further difference with the related literature mentioned is that we impose that prices cannot be negative. In particular, subsidizing the reception of calls is not allowed in our paper. Finally, we address the issue of multiplicity of equilibria in more detail by characterizing the full set of symmetric equilibria and by considering alternative selection methods.

We obtain the following results in the case of inelastic subscription demand. We show that equilibrium call and reception prices can still be found by assuming market shares constant. This implies that the possible equilibrium call and reception prices do not depend on the assumption of passive expectations. Fixed fees and profits do depend on this assumption when network-based price discrimination is considered. Further, if we introduce noise in the receivers’ utility in order to select a unique equilibrium we obtain that the known strategic marginal cost pricing principle generalizes to oligopolies. However, as we restrict prices to be non-negative, positive termination mark-ups will result in zero (rather than negative) reception charges. Hence, this can explain why countries with high termination charges exhibit CPP regimes while countries where termination rates are very low exhibit RPP regimes with strictly positive prices for receiving calls. We also show that the issue of connectivity breakdown is not severe when there are three or more firms, even when call externalities are very strong, so that callers and receivers obtain the same utility from a call of given length.

We also show that if utilities of receivers are not exposed to a random shock, there exists always an equilibrium in the RPP regime that resembles exactly the equilibrium under the CPP regime. That is, firms may play an equilibrium in which it is optimal to set reception charges equal to zero. This suggests that networks in Europe may stick to the CPP regime in order to make sure they will be able to coordinate on equilibrium play. It also implies that all the results obtained in Hurkens and López (2010) remain valid in the case where firms are allowed to charge consumers for receiving calls but just happen to charge them zero. As a final criterion for equilibrium selection we assume firms coordinate on the equilibrium that yields the highest profit. In the case of network-based price discrimination and industries with at least three firms this implies that firms will play the equilibrium in which callers

\[13\] There is profit neutrality among all equilibria when network-based price discrimination is not allowed.
and receivers jointly determine the volume of calls. In particular, when receivers and callers obtain the same utility from a call, the price for placing and receiving calls is the same in this equilibrium.

In order to be able to calculate, for any termination charge, equilibrium usage prices when subscription demand is elastic, we select again the equilibrium where callers and receivers jointly determine the length of the call. We show that both consumer and total surplus is maximized by the same termination mark-up $\bar{m} < 0$. The same termination mark-up also maximizes industry profit if and only if network externalities are extremely strong, that is when market penetration would be inefficiently low. If competition is effective in boosting penetration, firms would prefer to have lower or higher termination mark-ups. With lower termination mark-ups firms would make profit from origination while with higher termination mark-up they would earn more profit from termination.

In Hurkens and López (2010) we show that the way consumers form expectations about network sizes is crucial for the relationship between termination charges and equilibrium profit under the CPP regime. We observe that the intuition for the counter-intuitive results obtained by Laffont et al. (1998b), Gans and King (2001) and Berger (2005) relies on the assumption that consumers can correctly predict the size of each network, after any combination of prices. Consumers having such rationally responsive expectations means that any change of a price, how tiny it may be, by one firm is assumed to lead to an instantaneous rational change in expectations of all consumers, such that, given these changed expectations, optimal subscription decisions will lead realized and expected network sizes to coincide. So a unilateral change in price does not lead only to a change in market shares, but it also leads consumers to accurately predict how market shares will change. We propose to relax the assumption of rationally responsive expectations and to replace it by one of fulfilled equilibrium expectations. This concept was first introduced by Katz and Shapiro (1985). Basically, Katz and Shapiro (1985) assume that first consumers form expectations about network sizes, then firms compete, and finally consumers make optimal subscription or purchasing decisions, given the expectations. These decisions then lead to actual market shares and network sizes. Katz and Shapiro impose that, in equilibrium, realized and expected network sizes are the same. We will refer to such expectations throughout the paper as passive (self-fulfilled) expectations. They are passive as they do not respond to out of equilibrium deviations by firms. Hurkens and López (2010) show that this seemingly innocuous twist of the modeling of consumer expectations is able to reconcile the puzzle: When consumers have passive expectations, firms prefer termination charges above cost, and socially optimal termination charges are below or at cost (depending on the case that is under consideration).\(^\text{14}\)

\(^{14}\)Moreover, this result is robust to the inclusion of call externalities, an arbitrary number of mobile
It is worth mentioning that a few recent papers also attempt to reconcile the mentioned puzzle. Armstrong and Wright (2009)\textsuperscript{15}, Jullien, Rey and Sand-Zantman (2010)\textsuperscript{16}, and Hoernig, Inderst and Valletti (2009)\textsuperscript{17} have in common that they introduce additional realistic features of the telecommunication industry into the Laffont, Rey and Tirole (1998b) framework. They show that for some parameter range (and under rationally responsive expectations) joint profits increase as the termination charge increases above the cost. However, contrary to Hurkens and López (2010) these papers conclude that the need to regulate termination charges is reduced because the socially optimal termination charge would also be above cost.

Our paper proceeds as follows. Section 2 generalizes the model of JLT (2004) by considering a Logit formulation with any number of networks. It also defines the concept of passive expectations. We assume that the utility of receiving calls is proportional but smaller than the utility from placing calls. In section 3 we examine the case of inelastic subscription demand. We first describe the set of equilibria when the volume of calls is always determined by the same party (either caller or receiver), both for the case with and without network-based price discrimination. We then discuss the three equilibrium selection hypotheses and the optimal termination mark-ups from the firms’ point of view and from the social welfare point of view. Section 4 considers the case of inelastic subscription demand. Section 5 concludes. The appendix collects some of the lengthier proofs.

\section{The model}

We consider a general model of \(n \geq 2\) network operators. The \(n\) network operators have complete coverage and compete for a continuum of consumers of unit mass. We make the standard assumption of a balanced calling pattern, which means that the fraction of calls from a given subscriber of a given network and completed on another given (including the same) network is equal to the fraction of consumers subscribing to the terminating network.\textsuperscript{18}

\footnotesize{\textsuperscript{15}Armstrong and Wright (2009) argue that if MTM and FTM termination charges must be chosen uniformly, as is in fact the case in most European countries, firms will trade off desirable high FTM and desirable low MTM charges and arrive at some intermediate level, which may well be above cost.}

\footnotesize{\textsuperscript{16}Jullien, Rey and Sand-Zantman (2010) argue that the willingness to pay for subscription is related to the volume of calls. They introduce two types of users in the framework of ALRT: light users and heavy users. Light users only receive calls and are assumed to have an elastic subscription demand. Instead, full participation is assumed for heavy users, who can place calls and obtain a fixed utility from receiving calls.

\textsuperscript{17}Hoernig, Inderst and Valletti (2009) consider the existence of calling clubs so that the calling pattern is not uniform but skewed.

\textsuperscript{18}Dessein (2003, 2004) examines how unbalanced calling patterns between different customer types affect retail competition when network operators compete in the presence of the caller-pays regime.}
Cost structure. The fixed cost to serve each subscriber is \( f \), whereas \( c_O \) and \( c_T \) denote the marginal cost of providing a telephone call borne by the originating and terminating networks. The marginal cost of an on-net call is \( c \equiv c_O + c_T \). We let \( a \) denote the reciprocal access charge paid by the originating network to the terminating network.\(^\text{19}\) The termination mark-up is equal to:

\[
m \equiv a - c_T.
\]

The perceived cost of an off-net call for the originating network is the true cost \( c \) for on-net calls, augmented by the termination mark-up for the off-net calls: \( c_O + a = c + m \). The marginal cost of an off-net call for the terminating network is \( c_T - a = -m \).

Pricing. We consider competition among \( n \geq 2 \) networks. Each firm \( i \in N = \{1, 2, \ldots, n\} \) charges, in the most general setting, a tariff \( T_i = (F_i, p_i, r_i, \hat{p}_i, \hat{r}_i) \), consisting of a fixed fee \( F_i \), per-unit call and reception charges for on-net traffic \( (p_i \text{ and } r_i) \) and per-unit call and reception charges for off-net traffic \( (\hat{p}_i \text{ and } \hat{r}_i) \).\(^\text{20}\)

Individual demand. Subscribers obtain positive utility from making and receiving calls. The caller’s utility from making a call of length \( q \) minutes is \( u(q) \), whereas the receiver’s is \( \tilde{u}(q) \) from receiving a call of that length. \( u(\cdot) \) and \( \tilde{u}(\cdot) \) are twice continuously differentiable, increasing and concave. For tractability, we assume that

\[
\tilde{u}(q) = \beta u(q) \quad \text{with } 0 < \beta < 1.
\]

The caller’s demand function is given by \( u'(q(p)) = p \), whereas the receiver’s demand function is given by \( \tilde{u}'(\tilde{q}(r)) = r \). We consider the case in which callers and receivers can hang up. Therefore, the length of an on-net call is \( q(\max(p_i, r_i/\beta)) \), whereas the length of an off-net calls is \( q(\max(p_i, r_j/\beta)) \) in the absence of network-based price discrimination, and \( q(\max(p_i, \hat{r}_j/\beta)) \) in the presence of network-based price discrimination (for \( i \in N \) and \( j \in (N \setminus \{i\}) \)). In this setting there exist multiple equilibria, which we will derive in Sections 3 and 4. We also show below that by letting the receiver’s marginal utility be random, Jeon et al. (2004) single out one of them.

Market shares. We are interested in allowing for elastic subscription demand. We will use the Logit formulation, rather than the Hotelling model with hinterlands or a spokes model,\(^\text{19}\)Reciprocity means that a network pays as much for termination of a call on the rival network as it receives for completing a call originated on the rival network.\(^\text{20}\)When \( n \geq 3 \), it would be even more general to allow each firm to set different prices for off-net traffic depending on which network is being called or is calling. However, since attention will be restricted to symmetric equilibria we lose nothing from imposing that there cannot be discrimination between the prices set for traffic terminating or originating at different rival networks. This reduces the burden of notation.
because it can deal both with multiple firms and elastic subscription.\footnote{See Anderson and de Palma (1992) and Anderson, de Palma and Thisse (1992) for more details about the Logit model.}

Let $w_i$ be the value of subscribing to network $i$ (as defined below). Not subscribing at all yields constant utility $w_0$. Consumers have idiosyncratic tastes for each operator. So we add a random noise term $\varepsilon_i$ and define $U_i = w_i + \mu \varepsilon_i$. The parameter $\mu > 0$ reflects the degree of product differentiation in a Logit model. A high value of $\mu$ implies that most of the value is determined by a random draw so that competition between the firms is rather weak. The noise terms $\varepsilon_k$ are random variables of zero mean and unit variance, identically and independently double exponentially distributed. These terms reflect consumers’ preference for one good over another (they are known to the consumer but are unobserved by the firms). A consumer will subscribe to network $i \in N$ if and only if $U_i > U_j$ for all $j \in (N \setminus \{i\}) \cup \{0\}$. The probability of subscribing to network $i$ is denoted by $\alpha_i$. The probability to remain unsubscribed is denoted by $\alpha_0$. The probabilities are given by

$$\alpha_i = \frac{\exp[w_i/\mu]}{\sum_{k=0}^{n} \exp[w_k/\mu]}.$$  \tag{1}

It is easily verified that for all $i \in N \cup \{0\}$, $j \in N$, $t \in \{F,p,r,\hat{p},\hat{r}\}$

$$\frac{\partial \alpha_i}{\partial t_j} = \frac{\alpha_i(1 - \alpha_i)}{\mu} \frac{\partial w_i}{\partial t_j} - \frac{\alpha_i}{\mu} \sum_{k \in N \setminus \{i\}} \alpha_k \frac{\partial w_k}{\partial t_j}.$$  \tag{2}

A change in the fixed fee or the prices for on-net traffic of network $i$ does not affect the expected net surplus from subscribing to network $j \neq i$ (i.e. $\partial w_j/\partial t_i = 0$ for $t \in \{F,p,r\}$). It follows that for $i \in N$

$$\frac{\partial \alpha_i}{\partial F_i} = -\frac{\alpha_i(1 - \alpha_i)}{\mu},$$  \tag{3}

while for $j \in \{0,1,\ldots,n\} \setminus \{i\}$

$$\frac{\partial \alpha_j}{\partial F_i} = \frac{\alpha_i \alpha_j}{\mu}.$$  \tag{4}

Further,

$$\frac{\partial \alpha_i}{\partial p_i} = \frac{\alpha_i(1 - \alpha_i)}{\mu} \frac{\partial w_i}{\partial p_i}$$  \tag{5}

while for $j \in \{0,1,\ldots,n\} \setminus \{i\}$

$$\frac{\partial \alpha_j}{\partial p_i} = -\frac{\alpha_i \alpha_j}{\mu} \frac{\partial w_i}{\partial p_i}.$$  \tag{6}

Similarly,

$$\frac{\partial \alpha_i}{\partial r_i} = \frac{\alpha_i(1 - \alpha_i)}{\mu} \frac{\partial w_i}{\partial r_i}.$$  \tag{7}
while for $j \in \{0, 1, ..., n\}\{i\}$

$$\frac{\partial \alpha_j}{\partial p_i} = -\frac{\alpha_i \alpha_j}{\mu} \frac{\partial w_i}{\partial p_i}.$$  

(8)

On the other hand, off-net prices of network $i$ may affect the net utility from subscribing to a different network, through the effect on the volume of off-net calls.

**Consumer Surplus.** Consumer surplus in the Logit model has been derived by Small and Rosen (1981) as (up to a constant)

$$CS = \mu \ln \left( \sum_{k=0}^{n} \exp\left(\frac{w_k}{\mu}\right) \right) = w_0 - \mu \ln(\alpha_0),$$  

(9)

where the right-hand side follows from (1). Clearly, consumer surplus is increasing in market penetration $1 - \alpha_0$. In case one assumes that all consumers must subscribe to one of the networks (e.g., $w_0 = -\infty$), then

$$CS = \mu \ln \left( \sum_{k=1}^{n} \exp\left(\frac{w_k}{\mu}\right) \right) = w + \mu \ln n,$$

(10)

where the last equation holds in case of a symmetric solution where each network offers surplus $w_i = w$.

**Timing.** We assume that the terms of interconnection are negotiated or established by a regulator first. Then, for a given access charge $a$ (or equivalently, a given $m$) the timing of the game is the following:

1. Consumers form expectations about the number of subscribers of each network $i$: $\beta_i$.

   In a symmetric equilibrium, expected market sizes must be equal so that $\beta_0 \in [0, 1]$ is the number of consumers who do not subscribe to any network and $\beta_i = (1 - \beta_0)/n$ for all $i \in N$.

2. Firms take these expectations as given and choose simultaneously retail tariffs.

3. Consumers make rational subscription and consumption decisions, given their expectations and given the networks’ tariffs.

Therefore, market share $\alpha_i$ is a function of prices and consumer expectations. Self-fulfilling expectations imply that at equilibrium $\beta_i = \alpha_i$. 

11
3 Full participation

In this section we consider the case in which consumers always subscribe to one of the networks. They do not have an outside option. Hence, in this case \( \alpha_0 = \beta_0 = 0 \).

While in many European countries on-net/off-net price discrimination is a common practice, there are countries as for example the US where this is less common.\(^{22}\) In addition, in mobile telecommunications markets, it is not uncommon for firms to offer price plans in which both regimes (network-based and no network-based price discrimination) coexist. We will therefore analyze both the case where firms are allowed to discriminate between on-net and off-net traffic and where they are not.

As Jeon et al. (2004) point out, there exists a potential indeterminacy of equilibria. If call and reception charges are such that the caller determines the call volume (because \( q(p) < q(r/\beta) \)), then as the reception charge has no impact on volume, from the viewpoint of firms and subscribers only the sum \( \{F + rq\} \) matters, not its composition. Similarly, when the volume is determined by the receiver, only the sum \( \{F + pq\} \), and not its composition, matters. However, both call and reception charges matter in both cases. In what follows we construct the range of equilibria for all four cases: network-based and no network-based price discrimination and caller and receiver determined call volume. We shall establish that the so-called “strategic marginal cost pricing principle” generalizes to any number of firms and that, in equilibrium, it is independent of whether expectations about network sizes are passive or rationally responsive.

3.1 No network-based discrimination

In this section, we consider for a given reciprocal access charge \( a \) and consumer expectations \( \{\beta_i\}_{i \in N} \), competition under the receiver-pays regime and no network-based price discrimination.

**Caller determined call volume**

If callers determine the volume, in a symmetric equilibrium it must hold that \( r \leq \beta p \). Given the balanced calling pattern assumption and consumer expectations \( \{\beta_i\}_{i \in N} \) the surplus from

\(^{22}\)In the case of the US there is a technical reason: as the prefixes of mobile and fixed line numbers are not different, it is difficult for users to identify to which network the person being called belongs to.
subscribing to network \( i \) is given by

\[
\begin{align*}
    w_i &= u(q(p_i)) - p_i q(p_i) + \beta_i[\beta u(q(p_i)) - r_i q(p_i)] \\
    &+ \sum_{k \neq i} \beta_k[\beta u(q(p_k)) - r_i q(p_k)] - F_i.
\end{align*}
\]

Since consumers’ expectations are assumed passive we have that \( w_i \) is a function of expectations and prices, instead of market shares and prices as it is in the case of rationally responsive expectations. Note that

\[
\frac{\partial w_i}{\partial p_i} = -q(p_i) + \beta_i(\beta p_i - r_i)q'(p_i),
\]

while for \( j \neq i \)

\[
\frac{\partial w_j}{\partial p_i} = \beta_i(\beta p_i - r_j)q'(p_i).
\]

Since we will be interested in symmetric equilibria, let us fix \( p_j = p^*, r_j = r^*, \) and \( F_j = F^* \) for all \( j \neq i \).

Assuming that callers determine the volume, the profit of network \( i \) is

\[
\pi_i = \alpha_i [(p_i - c - m) q(p_i) + \alpha_i(r_i + m)q(p_i) + (1 - \alpha_i)(r_i + m)q(p^*) + F_i - f].
\]

One can calculate the optimal price for network \( i \) by keeping market shares constant by adjusting \( F_i \) appropriately, so as to keep the relative attractiveness \( w_i - w_j \) constant. This requires

\[
\frac{\partial F_i}{\partial p_i} = -q(p_i) + \beta_i(r^* - r_i)q'(p_i).
\]

The first-order condition yields

\[
p_i = c + (1 - \alpha_i)m + (\beta_i - \alpha_i)r_i - \beta_ir^*. \tag{11}
\]

When expectations are fulfilled, \( \alpha_i = \beta_i, \) so that \( p_i = c + (1 - \alpha_i)m - \alpha_ir^* \). Observe that the optimal call price depends on the reception charge (of the rivals). As the caller determines the volume, firms set the calling price at the perceived (or, as termed by JLT, “strategic marginal”) cost of placing a call, which is given by: the average marginal cost of a call \( c + (1 - \alpha_i)m \) minus the pecuniary externality imposed on the subscribers of the rival

13
network $\beta r^*$.\textsuperscript{23} In a symmetric equilibrium ($\alpha_i = 1/n$) Eq. (11) boils down to

$$p^* = c + \frac{(n-1)m - r^*}{n}.$$  \hfill (12)

When $p_j = p^*$ and $r_j = r^*$ for all $j \in N$, network $i$’s profit can be rewritten as follows

$$\pi_i = \alpha_i [(p^* + r^* - c) q(p^*) + F_i - f].$$  \hfill (13)

Using $\partial \alpha_i / \partial F_i = -\alpha_i (1 - \alpha_i) / \mu$, we obtain

$$\frac{d\pi_i}{dF_i} = -\frac{\alpha_i (1 - \alpha_i)}{\mu} ( (p^* + r^* - c) q(p^*) + F_i - f ) + \alpha_i.$$  

At symmetric equilibrium

$$F^* = f + \frac{n\mu}{n-1} - (p^* + r^* - c) q(p^*).$$  \hfill (14)

Any combination $(F^*, p^*, r^*)$ satisfying Eqs. (12) and (14), and $r^* \leq \beta p^*$ is an equilibrium (provided that no firm wants to deviate and set the reception charge above $\beta p^*$). By replacing Eqs. (12) and (14) into (13), we have that at symmetric equilibrium: $\pi = \frac{\mu}{n-1}$. That is, equilibrium profit is neutral to the access charge and equals the profit that firms would obtain under unit demand.

**Receiver determined call volume**

We now turn to the case in which receivers determine the volume. In equilibrium the condition $r \geq \beta p$ must hold. The surplus from subscribing to network $i$ is now given by:

$$w_i = \beta_i [u(q(r_i/\beta)) - p_i q(r_i/\beta)] + \beta q(q(r_i/\beta)) - r_i q(r_i/\beta)$$

$$+ \sum_{k \neq i} \beta_k [u(q(r_k/\beta)) - p_i q(r_k/\beta)].$$

Note that

$$\frac{\partial w_i}{\partial r_i} = -q(r_i/\beta) + \frac{\beta_i q'(r_i/\beta)}{\beta^2} (r_i - \beta p_i),$$

while

\textsuperscript{23}A decrease in $p_i$ will increase the volume of calls from network $i$ to network $j$. This in turn increases the surplus of network-$j$ subscribers from the calls they receive from network $i$ (direct externality), but also their payment as they have to pay for receiving these extra calls (pecuniary externality). Since the direct externality on $i$ and $j$’s subscribers is the same (the volume of calls received by consumers increases by the same amount independently of the network they are attached), only the pecuniary externality matters.
\[
\frac{\partial w_j}{\partial r_i} = \frac{\beta_i q'(r_i/\beta)}{\beta^2} (r_i - \beta p_j).
\]

Since we will be interested in symmetric equilibria, let us fix \(p_j = p^*, r_j = r^*, \) and \(F_j = F^*\) for all \(j \neq i\). The profit of network \(i\) is

\[
\pi_i = \alpha_i [(1 - \alpha_i) (p_i - c - m) q(r^*/\beta) + \alpha_i (p_i + r_i - c) q(r_i/\beta) + (1 - \alpha_i)(r_i + m) q(r_i/\beta) + F_i - f].
\]

(15)

One can calculate the optimal price for network \(i\) by keeping market shares constant by adjusting \(F_i\) appropriately, so as to keep the relative attractiveness \(w_i - w_j\) constant. This requires

\[
\frac{\partial F_i}{\partial r_i} = -q(r_i/\beta) + \frac{\beta_i q'(r_i/\beta)}{\beta} (p^* - p_i).
\]

The first-order condition yields:

\[
r_i = \alpha_i c - (1 - \alpha_i)m - \beta_i p^* + (\beta_i - \alpha_i)p_i.
\]

(16)

In a self-fulfilling equilibrium, \(\alpha_i = \beta_i\) so that \(r_i = \alpha_i c - (1 - \alpha_i)m - \alpha_i p^*\). Note that the optimal reception charge depends on the call price chosen by the rivals. When the receiver determines the volume, firm \(i\) sets the reception charge at the perceived (or strategic marginal cost) of receiving a call, which is given by: the average unit cost of receiving calls on a given network \(\alpha_i c - (1 - \alpha_i)m\) minus the pecuniary externality imposed on the subscribers of the rival network \(\alpha_i p\). At symmetric equilibrium, Eq. (16) reads as

\[
r^* = \frac{c - (n - 1)m - p^*}{n}.
\]

(17)

As above, it is straightforward to show that at the symmetric equilibrium

\[
F^* = f + \frac{n\mu}{n - 1} - (p^* + r^* - c) q(r^*/\beta).
\]

(18)

Any combination \((F^*, p^*, r^*)\) satisfying Eqs. (17) and (18), and \(r^* \geq \beta p^*\) is an equilibrium (provided that no firm wants to deviate and set the call price above \(r^*/\beta\)). By replacing Eqs. (17) and (18) into (15), we have that at symmetric equilibrium: \(\pi = \frac{\mu}{n - 1}\). That is, equilibrium profit is neutral to the access charge and equals the profit that firms would obtain under unit demand.

Fig. 1 illustrates the set of symmetric equilibria. The point \(X\) is the intersection point of Eqs. (12) and (17). When \(m > -\beta c/(1 + \beta)\), \(X\) lies in the region where the caller determines the volume. When \(m < -\beta c/(1 + \beta)\), \(X\) lies in the region where the receiver determines the
volume. Note that only the thick parts represent equilibria. For example, the point $Y$ is not an equilibrium since networks would have an incentive to raise the reception charge above $\beta p$ (in Fig. 1(a)) and to raise the call price above $r/\beta$ (in Fig. 1(b)).

In order to formalize our findings in a proposition we introduce some notation. Let

$$\bar{m} = \frac{-\beta c}{1+\beta}.$$

For $m \geq \bar{m}$ let $(\bar{p}(m), \bar{r}(m))$ denote the intersection point of the line $r = \beta p$ with first-order condition (12). For $m < \bar{m}$ let $(\bar{p}(m), \bar{r}(m))$ denote the intersection point of the line $r = \beta p$ with first-order condition (17).

**Proposition 1** The set of symmetric equilibria in which the caller determines the volume of calls is given by

$$E_C = \{(p^*, r^*, F^*) : p^* \geq \bar{p}(m), r^* \geq 0, \text{ Eq. (12), Eq. (14)}\}.$$

The set of symmetric equilibria in which the receiver determines the volume of calls is given by

$$E_R = \{(p^*, r^*, F^*) : p^* \geq 0, r^* \geq \bar{r}(m), \text{ Eq. (17), Eq. (18)}\}.$$

In any symmetric equilibrium, equilibrium profit equals $\pi^* = \mu/(n-1)$.

![Figure 1: Equilibrium prices: No network-based price discrimination.](image)

(a) $m > -\beta c/(1 + \beta)$

(b) $m < -\beta c/(1 + \beta)$
3.2 Network-based discrimination

When the market for on-net traffic is distinguished from the market for off-net traffic, firms will want to set on-net call and reception charges so as to maximize the utility obtained from on-net traffic by internalizing the call externality. This can be arranged by a number of combination of such prices. Namely, optimality requires that the volume of on-net traffic \( q \) satisfies \((1+\beta)u'(q) = c\). This can be obtained either by having callers determine the volume \( (p_i = c/(1+\beta) \) and \( r_i \leq \beta p_i \)) or by having receivers determine the volume \( (r_i = \beta c/(1+\beta) \) and \( p_i \leq r_i/\beta \)). If there would be some very small and vanishing noise in the receiver’s or caller’s utilities then the optimal prices would converge to

\[
(p^*, r^*) = \left( \frac{c}{1+\beta}, \frac{\beta c}{1+\beta} \right).
\]

For these prices the caller and receiver jointly determine the volume of calls. Without loss of generality we will restrict attention to equilibria with on-net prices equal to \( p^* \) and \( r^* \). Note that \( p^* + r^* = c \) in this case so that no profit is obtained from on-net traffic (except for the fixed fee that is levied on subscribers).

Determining the prices for off-net traffic is more complicated since these prices affect consumers on other networks. Even if one focuses on symmetric equilibria in which the caller determines the volume of calls (so that \( \hat{p}_i > \hat{r}_j/\beta \)), the actual level of \( \hat{r}_j \) is important to determine the optimal \( \hat{p}_i \). Similarly, even if one focuses on symmetric equilibria in which the receiver determines the volume of calls (so that \( \hat{p}_i < \hat{r}_j/\beta \)), the actual level of \( \hat{p}_i \) is important to determine the optimal \( \hat{r}_j \). We will analyze these equilibria in turn in the next subsections.

**Caller determined call volume**

We fix \( \hat{r}_i = \hat{r}^* \) for all \( i \in N \). We look for a symmetric equilibrium. Since subscription demand is assumed inelastic and the off-net call price \( \hat{p}_i \) will affect all rivals in the same way (in a symmetric equilibrium), one can actually calculate the optimal off-net call price of network \( i \) by keeping market shares constant at \( 1/n \) (by adjusting \( F_i \) accordingly). Fixing \( \hat{p}_j = \hat{p}^* \) and \( F_j = F^* \) for all \( j \neq i \), the profit of network \( i \) is equal to

\[
\pi_i = \alpha_i((1-\alpha_i)(\hat{p}_i - c - m)q(\hat{p}_i) + (1-\alpha_i)(\hat{r}^* + m)q(\hat{p}^*) + F_i - f),
\]
where $F_i$ is such that $w_i = w_j$, that is

$$F_i = \frac{n-1}{n}[(u(q(\hat{p}_i)) - \hat{p}_i q(\hat{p}_i)) - (u(q(\hat{p}^*)) - \hat{p}^* q(\hat{p}^*))]
+ \frac{1}{n}[(\beta u(q(\hat{p}^*)) - \hat{r}^* q(\hat{p}^*)) - (\beta u(q(\hat{p}_i)) - \hat{r}^* q(\hat{p}_i))] + F^*$$

Observe that

$$\frac{\partial F_i}{\partial \hat{p}_i} = \frac{n-1}{n}[-q(\hat{p}_i)] - \frac{1}{n}[(\beta \hat{p}_i - \hat{r}^*) q'(\hat{p}_i)].$$

At a symmetric equilibrium (with market share $\alpha_i$ kept constant at $1/n$) $\hat{p}_i$ must satisfy

$$0 = \frac{\partial \pi_i}{\partial \hat{p}_i} = \frac{1}{n} \left[ \frac{n-1}{n} [q(\hat{p}_i) + (\hat{p}_i - c - m) q'(\hat{p}_i) - q(\hat{p}_i)] - \frac{1}{n} (\beta \hat{p}_i - \hat{r}^*) q'(\hat{p}_i) \right]
= \frac{q'(\hat{p}_i)}{n^2} \left[ (n - 1 - \beta) \hat{p}_i - (n - 1)(c + m) + \hat{r}^* \right]$$

so that

$$(n - 1 - \beta) \hat{p}_i - (n - 1)(c + m) + \hat{r}^* = 0.$$ 

Note that the second-order derivative of profits, evaluated at the solution of the first-order condition, reads

$$\frac{\partial^2 \pi}{\partial \hat{p}_i^2} = \frac{q'(\hat{p}_i)}{n^2} (n - 1 - \beta) < 0$$

for all $\beta < 1$ and $n \geq 2$. Hence, in a symmetric equilibrium in which callers determine the volume of calls, we must have

$$\hat{p}^* = \frac{(n - 1)(c + m) - \hat{r}^*}{n - 1 - \beta}$$

and $0 \leq \hat{r}^* \leq \beta \hat{p}^*$ or, equivalently, $0 \leq \hat{r}^* \leq \beta(c + m)$. Substituting these prices into the profit function yields

$$\pi_i = \alpha_i((1 - \alpha_i)(\hat{p}^* + \hat{r}^* - c)q(\hat{p}^*) + F_i - f).$$

To find the equilibrium fixed fee we solve the first-order condition

$$0 = \frac{\partial \pi}{\partial F_i} = -\frac{\alpha_i(1 - \alpha_i)}{\mu} ((1 - 2\alpha_i)(\hat{p}^* + \hat{r}^* - c)q(\hat{p}^*) + F_i - f) + \alpha_i.$$ 

At a symmetric equilibrium $\alpha_i = 1/n$ so that

$$F^* = f + \frac{\mu}{n} - \frac{2}{n} (\hat{p}^* + \hat{r}^* - c)q(\hat{p}^*),$$ (20)
and the equilibrium profit equals

$$\pi^* = \frac{\mu}{n - 1} + \frac{1}{n^2}(\hat{p}^* + \hat{r}^* - c)q(\hat{p}^*).$$

(21)

Note, however, that condition (19) is necessary but not quite sufficient. In particular, one needs to check whether a network has an incentive to raise the reception charge for off-net calls above $\beta \hat{p}^*$. The next lemma addresses this question.

**Lemma 2** A necessary condition for $(\hat{p}^*, \hat{r}^*)$ as defined in (19) to be off-net usage prices in a symmetric equilibrium in which the caller determines call volume is that $\beta \hat{p}^* \geq -m$. It is sufficient if moreover $(n - 1)\beta > 1$. Otherwise provoking connectivity breakdown by setting $\hat{r}_i^* = \infty$ may be profitable.

Notice that the number of firms is important when one wants to check whether a firm has an incentive to create connectivity breakdown by raising the off-net reception price to infinite. If the number of firms is high (so that $\beta > 1/(n - 1)$) there is no incentive to do this. The intuition is that a connectivity breakdown provoked by network $i$ will affect the subscribers of other networks only with respect to the calls made to subscribers of network $i$, which is only a fraction $1/(n - 1)$ of all off-net calls made. On the other hand, the subscribers of network $i$ will not be able to receive any off-net call. As long as $\beta > 1/(n - 1)$ a connectivity breakdown hurts subscribers from network $i$ more than those of rival networks. In the case of duopoly, creating connectivity breakdown may be optimal. For example, let $n = 2$ and $m \leq 0$ and consider the equilibrium candidate $(\hat{p}, \hat{r}) = ((c + 2m)/(1 - \beta), -m)$. (This is in fact the equilibrium candidate selected by JLT (2004) as noise in the receiver’s utility vanishes.) In this case no profit is made from receiving off-net calls. By deviating to $\hat{r}_1 = \infty$ network 1 hurts subscribers on the rival network more than its own subscribers, if the call externality is relatively weak. As a result, it can raise the fixed fee to its own subscribers such that it still attracts half of the market. This raises the profit of network 1. On the other hand, let for example $m = 0$ and consider the equilibrium candidate $(\hat{p}, \hat{r}) = (c, \beta c)$. Recall that equilibrium (candidate) profit equals $\mu + \frac{1}{4} \beta c q(c)$. Then no profit is made on calls placed off-net but profit is made on calls received off-net. Provoking connectivity breakdown now will imply losing the latter profits. In fact, profit will equal $\pi_1 = \alpha_1(F_1 - f)$. The optimal fixed fee satisfies thus $F_1 - f = \mu/(1 - \alpha_1)$. To keep market share constant at 1/2, network 1 can raise its fixed fee by $\frac{1}{2}(1 - \beta)v(c)$ where $v(c) = u(q(c)) - cq(c)$. It will be optimal to raise its fixed fee by a little less and increase market share above 1/2. In any case, for $\beta$ sufficiently close to 1 the raise in fixed fee and market share will not be sufficient to compensate the loss of the profits from receiving off-net calls.
Receiver determined call volume

We fix \( \hat{p}_i = \hat{p}^* \) for all \( i \in N \). We look for a symmetric equilibrium. Since subscription demand is assumed inelastic and the off-net reception price \( \hat{r}_i \) will affect all rivals in the same way (in a symmetric equilibrium), one can actually calculate the optimal off-net reception price of network \( i \) by keeping market shares constant at \( 1/n \) (by adjusting \( F_i \) accordingly). Fixing \( \hat{r}_j = \hat{r}^* \) and \( F_j = F^* \) for all \( j \neq i \), the profit of network \( i \) is equal to

\[
\pi_i = \alpha_i((1 - \alpha_i)(\hat{p}^* - c - m)q(\hat{r}_i^*/\beta) + (1 - \alpha_i)(\hat{r}_i + m)q(\hat{r}_i/\beta) + F_i - f),
\]

where \( F_i \) is such that \( w_i = w_j \), that is

\[
F_i = \frac{n-1}{n} \frac{1}{\beta} \frac{1}{n} \left[ (\beta \left( \frac{n-1}{n} \right)) - \frac{1}{\beta} \left( \beta \left( \frac{n-1}{n} \right) \right) \right] + \frac{1}{n} \left( \frac{u(\hat{r}_i^*/\beta)}{\beta} - \frac{1}{\beta} \left( \beta \left( \frac{n-1}{n} \right) \right) \right) + F^*.
\]

Observe that

\[
\frac{\partial F_i}{\partial \hat{r}_i} = \frac{n-1}{n} \left[ - \frac{1}{\beta} \left( \beta \left( \frac{n-1}{n} \right) \right) \right] + \frac{1}{n} \left( \frac{u(\hat{r}_i^*/\beta)}{\beta} - \frac{1}{\beta} \left( \beta \left( \frac{n-1}{n} \right) \right) \right) + F^*.
\]

At a symmetric equilibrium (with market share \( \alpha_i \) kept constant at \( 1/n \)) \( \hat{r}_i \) must satisfy

\[
0 = \frac{\partial \pi_i}{\partial \hat{r}_i} = \frac{n-1}{n} \left[ \frac{1}{\beta} \left( \beta \left( \frac{n-1}{n} \right) \right) \right] + \frac{1}{n} \left( \frac{u(\hat{r}_i^*/\beta)}{\beta} - \frac{1}{\beta} \left( \beta \left( \frac{n-1}{n} \right) \right) \right) + F^*.
\]

so that

\[
(\beta(n-1) - 1)\hat{r}_i + \beta(n-1)m + \beta \hat{p}^* = 0.
\]

Hence, in a symmetric equilibrium in which receivers determine the volume of calls, we must have \( 0 \leq \hat{p}^* \leq \hat{r}^* \) and

\[
\hat{r}^* = \frac{\beta((n-1)m + \hat{p}^*)}{1 - (n-1)\beta}.
\]

Note that at the solution of the first-order condition, the second order derivative reads

\[
\frac{\partial^2 \pi}{\partial \hat{r}_i^2} = \frac{\beta^2(n-1)}{\beta^2 n^2} (\beta(n-1) - 1),
\]

which is negative only if \( \beta > 1/(n-1) \). For example, for \( n = 2 \) and \( \beta < 1 \) the second-order derivative is positive. This means that in this case, the equilibrium candidate is not an equilibrium. The only symmetric equilibrium in which the receiver determines the volume
of calls is then where \( \hat{r} = \infty \), that is where no calls are made off-net. If \( (n - 1)\beta - 1 > 0 \), an equilibrium in which receivers determine the volume of calls may exist. A necessary condition is that \( m < 0 \), \( \hat{p}^* \in [0, -m/\beta] \) and \( \hat{r}^* \) as above.

In the equilibrium candidate we have, as before

\[
F^* = f + \frac{n\mu}{n - 1} - \frac{n - 2}{n} (\hat{p}^* + \hat{r}^* - c)q(\hat{r}^*/\beta),
\]

so that the equilibrium profit equals

\[
\pi^* = \frac{\mu}{n - 1} + \frac{1}{n^2} (\hat{p}^* + \hat{r}^* - c)q(\hat{r}^*/\beta).
\]

However, condition (22) is not yet quite sufficient, as one needs to check whether a firm \( i \) would like to deviate and set \( \hat{p}_i > \hat{r}^*/\beta \). The next lemma addresses this question.

**Lemma 3** A necessary condition for \( (\hat{p}^*, \hat{r}^*) \) as defined in (22) to be off-net usage prices in a symmetric equilibrium in which the receiver determines call volume is that \( (n - 1)\beta > 1 \) and that \( \hat{r}^* \geq \beta(c + m) \).

The next figure illustrates the set of equilibrium (candidate) off-net usage prices. As shown before, equilibria in which the receiver determines call volume may only exist when \( (n - 1)\beta > 1 \). Moreover, if \( (n - 1)\beta < 1 \), the equilibrium candidates where the caller determines call volume may be vulnerable to deviations in which a network sets reception price equal to \( \infty \).

![Figure 2: Equilibrium off-net prices.](image)

(a) \( m > -\beta c/(1 + \beta) \)  
(b) \( m < -\beta c/(1 + \beta) \)
3.3 Equilibrium selection

We have shown that even when restricting attention to symmetric equilibria, a multiplicity of equilibria exists in all cases. This means that networks face a huge coordination problem. Without addressing this coordination problem it is very difficult to do policy analysis and recommend a particular termination mark-up. Namely, the socially optimal termination mark-up will depend on which of the equilibria networks will play. We discuss briefly three possible equilibrium selection hypotheses. First, we can consider using the CPP regime as an equilibrium selection device where the equilibrium with zero reception charges is used. Second, by introducing (vanishing) noise in the marginal utilities of receivers one can ensure that both callers and receivers sometimes determine the length of the call, so that two first-order conditions must be satisfied simultaneously. This selects a unique equilibrium. Finally, we can consider that firms choose the equilibrium that maximizes their profits. And in case of indifference they play the one that maximizes consumer (and thus total) surplus.

3.3.1 CPP as a selection device

In the case of no network-based price discrimination this means that firms play the equilibrium

\[ r^* = 0, p^* = c + \frac{n-1}{n}m, F^* = f + \frac{n\mu}{n-1} + (p^* - c)q(p^*). \]

As observed before, profit is neutral with respect to termination mark-up. The socially optimal termination mark-up would be the one that achieves the efficient call volume, i.e. such that \( p^* = c/(1+\beta) \). Hence, the optimal termination mark-up would be

\[ m^W = \left(\frac{-\beta c}{1+\beta}\right) \left(\frac{n}{n-1}\right), \]

if \( m^W > -c_T \) and \(-c_T \) otherwise. So for sufficiently strong call externality, Bill and Keep will be optimal.

In the case of network-based price discrimination this means that firms play the equilibrium with

\[ r^* = 0, p^* = \frac{c}{1+\beta}, \hat{r}^* = 0, \hat{p}^* = (c + m) \frac{n-1}{n-1-\beta}. \]

Note that if \( n = 2 \), the off-net call price goes to infinity as the call externality \( \beta \rightarrow 1 \). In this sense the equilibrium exhibits asymptotic connectivity breakdown. However, this is not true if there are at least three firms.

As observed before, profit is not neutral with respect to termination mark-up. The termination mark-up that maximizes firms’ profits is the one that maximizes \((\hat{p}^* - c)q(\hat{p}^*)\).
That is, firms prefer termination mark-up that yields off-net price equal to monopoly price \( p^M \). Hence, the termination mark-up that maximizes firms’ profits equals

\[
m^\pi = \frac{n - 1 - \beta}{n - 1} p^M - c.
\]

Note that for a duopoly market, the optimal termination mark-up is negative if the call externality is strong enough. However, for industries with at least three firms the profit maximizing termination mark-up will be strictly positive, if call demand is relatively inelastic.

The socially optimal termination mark-up would be the one that achieves the efficient call volume, i.e. such that \( \hat{p}^* = c/(1 + \beta) \). Hence, the optimal termination mark-up would be

\[
m^W = \left( \frac{-\beta c}{1 + \beta} \right) \left( \frac{n}{n - 1} \right),
\]

if \( m^W > -c_T \) and \(-c_T \) otherwise. So for sufficiently strong call externality, Bill and Keep will be optimal.

### 3.3.2 Noisy receiver utility

This approach has been introduced in JTL (2004) and López (2010). We do not replicate their analysis here but summarize the findings. By assuming that the utility of the receiver is subject to noise sometimes the caller and sometimes the receiver will hang up first. That is, both parties determine the volume of calls.

In the case where network-based price discrimination is not allowed, it turns out that the equilibrium prices are found at the intersection of Eqs. (12) and (17). That is, the equilibrium is at the point \( X \) indicated in Fig. 1. It is easily established that this equilibrium has

\[
p = c + m, r = -m,
\]

as long as \( m \leq 0 \). For \( m > 0 \) the equilibrium prices would be \( p = c + m/2 \) and \( r = 0 \), as negative prices are not allowed even though firms would like to subsidize incoming calls. Profit is neutral with respect to termination mark-up. The socially optimal termination mark-up in this case is equal to

\[
m^W = \frac{-\beta c}{1 + \beta},
\]

if \( m^W > -c_T \), otherwise it is equal to \(-c_T \). In particular, for \( \beta = 1 \) and if \( c_O = c_T \), Bill and Keep is socially optimal.

In the case where network-based price discrimination is allowed, JTL (2004) find the following equilibrium candidate as the noise vanishes, for the duopoly case, when \(-\beta c/(1 +\)
$\beta < m < 0$:

$$p^* = \frac{c}{1 + \beta}, r^* = \frac{\beta c}{1 + \beta}, \hat{p}^* = \frac{c + 2m}{1 - \beta}, \hat{r}^* = -m.$$  

For $m \geq 0$ the equilibrium has $\hat{r}^* = 0$ and $\hat{p}^* = (c + m)/(1 - \beta)$.

The termination mark-up that maximizes firms’ profits in this equilibrium candidate is the one that maximizes $(\hat{p}^* + \hat{r}^* - c)q(\hat{p}^*)$. It is not straightforward to determine the optimal termination mark-up for general call demand functions. However, under the additional assumption of constant elasticity call demand $q(p) = p^{-\eta}$ one can show that firms prefer a negative termination mark-up if and only if the call externality is sufficiently strong in relation to the elasticity of call demand. The socially optimal termination mark-up would be the minimal one

$$m_W = \frac{-\beta c}{1 + \beta}.$$  

**Remark 4** According to JLT (2004) this equilibrium candidate does not exist for $m = -\beta c/(1 + \beta)$ so that the socially optimum cannot be reached. More importantly, though, as we have demonstrated before, the equilibrium candidate is not an equilibrium for any $m < 0$ when there are only two firms. Recall from our earlier discussion that each firm has an incentive to cause connectivity breakdown by raising the reception charge to infinite. However, we do conjecture that noise will select the equilibrium candidate with $\hat{r} = -m$ for any number of firms and that, when there are at least three firms, the candidate is in fact an equilibrium. Namely, in a symmetric equilibrium with at least three firms there is no incentive to cause connectivity breakdown as it hurts subscribers from rival networks only partially (at most half of the calls received from other networks) while it hurts subscribers of the own network fully (it affects all calls placed off-net). We further conjecture that, if the number of firms is large enough, the termination mark-up that maximizes profit is always positive.

### 3.3.3 Perfect coordination

Assuming that firms can perfectly coordinate they would play the equilibrium with the highest profit. Note that this does not involve perfect collusion since firms do set prices in a competitive fashion. As we have argued before, if there are at least three firms, the profit maximizing equilibrium is the one where $\beta \hat{p}^* = \hat{r}^*$ as this maximizes the volume of calls and the sum of call and reception price. Hence, for $n \geq 3$ firms will play the equilibrium with

$$p^* = \frac{c}{1 + \beta}, r^* = \frac{\beta c}{1 + \beta}, \hat{p}^* = c + m, \hat{r}^* = \beta(c + m),$$

if $m \geq -\beta c/(1 + \beta)$. 

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Again, it is not possible to determine the optimal termination mark-up for general call
demand function. However, under the assumption of constant elasticity call demand \( q(p) = p^{-\eta} \), one can show that profit maximizing termination mark-up equals

\[
m^* = \frac{c(1 + \beta(1 - \eta))}{(1 + \beta)(\eta - 1)}.
\]

The socially optimal termination mark-up, under the assumption that firms coordinate
on their best equilibrium has again

\[
m^W = \frac{-\beta c}{1 + \beta}.
\]

4 Elastic subscriber participation with network-based
discrimination

We will now consider the case where participation by consumers is voluntary. That is,
consumers have an outside option of not subscribing to any of the networks. They will only
subscribe to a network if it yields at least the value of the outside option. This allows us
to address the question how the payment regime of RPP affects penetration. It has been
argued that RPP regimes lead to lower participation but the empirical evidence is not clear
while no theoretical model has been developed to address this issue thus far.

Analyzing partial participation under call externalities and the RPP regime is very chal-
lenging since it is not straightforward to determine the usage prices for placing and receiving
calls. This is due to the fact that, for example, an increase in the call price of one network
lowers the surplus of subscribing to any of the networks. The network can adjust its fixed
fee in order to keep the number of its own subscribers constant, but it cannot avoid that the
overall penetration goes down. Hence, in general it is not possible to adjust the fixed fee
to keep all market shares constant. This in turn implies that one cannot maximize profits
assuming that market shares stay constant. However, there is one exception. When \( \beta \hat{p}_i = \hat{r}_j \)
a marginal increase in \( \hat{p}_i \) will not affect the surplus from subscribing to network \( j \) since the
utility lost from receiving less calls is exactly compensated by the reduction in reception
payments. Therefore we will focus on equilibria where \( \beta \hat{p} = \hat{r} \). In these equilibria in some
sense both the caller and the receiver determine the call volume. Namely, if either the call or
reception price is marginally increased volume is reduced. However, if either price is reduced
volume remains the same.

Notice that networks will set on-net prices efficiently. This requires \( p_i \leq c/(1 + \beta) =: p^* \)
and \( r_i \leq \beta c / (1 + \beta) =: r^* \) with at least one equality. Without loss of generality we assume that both inequalities are binding. In this way profits stem only from the fixed fee, from off-net calls and from termination service. We need to determine fixed fees and off-net usage prices.

### 4.1 Off-net usage prices

We will first focus on symmetric equilibria where the call price \( \hat{p}^* \) satisfies the first-order condition and where \( \hat{r}^* = \beta \hat{p}^* \). Hence we need that \( \partial \pi_i / \partial \hat{p}_i = 0 \) and \( \partial \pi_i / \partial \hat{r}_i \leq 0 \). Let \( \bar{\beta} \) denote the expected number of subscribers for each network. Clearly, \( \bar{\beta} < 1/n \). Let \( \hat{p}_j = \hat{p}^* \) and \( F_j = F^* \) for all \( j \neq i \) and \( \hat{r}_j = \hat{r}^* = \beta \hat{p}^* \) for all \( j \). Consider call price \( \hat{p}_i \) of network \( i \) such that \( \hat{p}_i \geq \hat{p}^* \). Now the surplus from subscribing to network \( i \) equals

\[
\begin{align*}
    w_i &= \bar{\beta}((1 + \beta)u(q(p^*)) - (p^* + r^*)q(p^*)) \\
    &+ (n - 1)\bar{\beta}(u(q(\hat{p}_i)) - \hat{p}_i q(\hat{p}_i)) + (n - 1)\bar{\beta}(\beta u(q(\hat{p}^*)) - \hat{r}^* q(p^*)) - F_i.
\end{align*}
\]

The first term represents the utility from placing and receiving on-net calls, the second and third term represent the utility from placing and receiving, respectively, off-net calls and the final term is the fixed fee. Similarly, the surplus from subscribing to network \( j \neq i \) equals

\[
\begin{align*}
    w_j &= \bar{\beta}((1 + \beta)u(q(p^*)) - (p^* + r^*)q(p^*)) \\
    &+ (n - 2)\bar{\beta}((1 + \beta)u(q(\hat{p}^*)) - (\hat{p}^* + \hat{r}^*)q(\hat{p}^*)) \\
    &+ \bar{\beta}(u(q(\hat{p}^*)) - \hat{p}^* q(\hat{p}^*)) + \bar{\beta}(\beta u(q(\hat{p}_i)) - \hat{r}^* q(\hat{p}_i)) - F^*.
\end{align*}
\]

The first term represents the utility from placing and receiving on-net calls, the third and fourth term represent the utility from placing and receiving, respectively, calls to and from subscribers of network \( i \), while the second term represents the utility from placing and receiving other calls. The last term is again the fixed fee. Note that

\[
\frac{\partial w_i}{\partial \hat{p}_i} = -(n - 1)\bar{\beta}q(\hat{p}_i)
\]

and

\[
\frac{\partial w_j}{\partial \hat{p}_i} = \bar{\beta}(\beta \hat{p}_i - \hat{r}^*)q'(\hat{p}_i).
\]

In particular, evaluated at \( \hat{p}_i = \hat{p}^* \) we have \( \partial w_j / \partial \hat{p}_i = 0 \). By adjusting \( F_i \) appropriately network \( i \) can maximize profits with respect to \( \hat{p}_i \) and keep number of subscribers constant.
at $\alpha_i$ while also keeping the number of subscribers of other networks constant. This requires
\[
\frac{\partial F_i}{\partial \hat{p}_i} = -(n-1)\bar{\beta}q(\hat{p}_i).
\]

Profit equals
\[
\pi_i = \alpha_i ((1 - \alpha_i - \alpha_0)[(\hat{p}_i - c - m)q(\hat{p}_i) + (\hat{r}^* + m)q(\hat{r}^*)] + F_i - f).
\]

Maximizing profits with respect to $\hat{p}_i$ while maintaining the number of subscribers to all networks constant by adjusting the fixed fee yields
\[
0 = \frac{\partial \pi_i}{\partial \hat{p}_i} = \alpha_i((1 - \alpha_i - \alpha_0)(q(\hat{p}_i) + (\hat{p}_i - c - m)q'(\hat{p}_i)) - (n-1)\bar{\beta}q(\hat{p}_i)).
\]

At a symmetric equilibrium with fulfilled expectations $\hat{p}_i = \hat{p}^*$ and $\alpha_j = \bar{\beta} =: \bar{\alpha}$ for all $j \in N$ and $\alpha_0 = 1 - n\bar{\alpha}$ so that we must have
\[
0 = \bar{\alpha}(n-1)(\hat{p}^* - c - m)q'(\hat{p}^*).
\]

Hence, the usage prices at the symmetric equilibrium satisfy
\[
\hat{p}^* = c + m
\] (25)

and
\[
\hat{r}^* = \beta(c + m).
\] (26)

Note that $\hat{p}^* \geq c/(\beta + 1)$ or, equivalently, $\hat{p}^* + \hat{r}^* \geq c$ if and only if $m \geq -\beta c/(\beta + 1)$. We will show later that this condition is equivalent to $\partial \pi_i/\partial \hat{r}_i \leq 0$ when evaluated at the equilibrium candidate $(\hat{p}^*, \hat{r}^*)$. The latter is a necessary condition for the equilibrium candidate to be an equilibrium.

We will next focus on symmetric equilibria where the reception price $\hat{r}^*$ satisfies the first-order condition and where $\hat{r}^* = \beta \hat{p}^*$. Hence we need that $\partial \pi_i/\partial \hat{r}_i = 0$ and $\partial \pi_i/\partial \hat{p}_i \leq 0$. Let again $\bar{\beta}$ denote the expected number of subscribers for each network. Let $\hat{r}_j = \hat{r}^*$ and $F_j = F^*$ for all $j \neq i$ and $\hat{p}_j = \hat{p}^* = \hat{r}^*/\beta$ for all $j$. Consider reception price $\hat{r}_i$ of network $i$ such that $\hat{r}_i \geq \hat{r}^*$. In this case the surplus from subscribing to network $i$ equals
\[
w_i = \bar{\beta}((1 + \beta)u(q(\hat{p}^*)) - (p^* + r^*)q(p^*)) + (n-1)\bar{\beta}(u(q(\hat{p}^*)) - \hat{p}^*q(\hat{p}^*)) + (n-1)\bar{\beta}(\beta u(q(\hat{r}_i/\beta)) - \hat{r}_i q(\hat{r}_i/\beta)) - F_i.
\]
The first term represents the utility from placing and receiving on-net calls, the second and third term represent the utility from placing and receiving, respectively, of off-net calls and the final term is the fixed fee. Similarly, the surplus from subscribing to network \( j \neq i \) equals

\[
w_j = \tilde{\beta}((1 + \beta)u(q(p^*)) - (p^* + r^*)q(p^*)) + (n - 2)\tilde{\beta}((1 + \beta)u(q(\hat{p}^*)) - (\hat{p}^* + \hat{r}^*)q(\hat{p}^*)) + \tilde{\beta}(u(q(\hat{r}_i/\beta)) - \hat{p}^* q(\hat{r}_i/\beta)) + \tilde{\beta}(\beta u(q(\hat{p}^*)) - r^* q(\hat{p}^*)) - F^*.
\]

The first term represents the utility from placing and receiving on-net calls, the third and fourth term represent the utility from placing and receiving calls to and from subscribers of network \( i \), while the second term represents the utility from placing and receiving other calls. The last term is again the fixed fee. Note that

\[
\frac{\partial w_i}{\partial \hat{r}_i} = -(n - 1)\tilde{\beta} q(\hat{r}_i/\beta)
\]

and

\[
\frac{\partial w_j}{\partial \hat{r}_i} = (\tilde{\beta}/\beta)(\hat{r}_i/\beta - \hat{p}^*) q'(\hat{r}_i/\beta).
\]

In particular, evaluated at \( \hat{r}_i = \hat{r}^* \) we have \( \partial w_j/\partial \hat{r}_i = 0 \). By adjusting \( F_i \) appropriately network \( i \) can maximize profits with respect to \( \hat{r}_i \) and keep number of subscribers constant at \( \alpha_i \) while also keeping the number of subscribers of other networks constant. This requires

\[
\frac{\partial F_i}{\partial \hat{r}_i} = -(n - 1)\tilde{\beta} q(\hat{r}_i/\beta).
\]

Profit equals

\[
\pi_i = \alpha_i ((1 - \alpha_i - \alpha_0)(\hat{p}^* - c - m) q(\hat{p}^*) + (\hat{r}_i + m) q(\hat{r}_i/\beta)] + F_i - f).
\]

Maximizing profits with respect to \( \hat{r}_i \) while maintaining the number of subscribers to all networks constant by adjusting the fixed fee yields

\[
0 = \frac{\partial \pi_i}{\partial \hat{r}_i} = \alpha_i((1 - \alpha_i - \alpha_0)(q(\hat{r}_i/\beta) + (\hat{r}_i + m) q'(\hat{r}_i/\beta)/\beta) - (n - 1)\tilde{\beta} q(\hat{r}_i/\beta)).
\]

At a symmetric equilibrium with fulfilled expectations \( \hat{r}_i = \hat{r}^* \) and \( \alpha_j = \tilde{\beta} =: \bar{\alpha} \) for all \( j \in N \) and \( \alpha_0 = 1 - n\bar{\alpha} \) so that we must have

\[
0 = \bar{\alpha}^2(n - 1)[\hat{r}^* + m] q'(\hat{r}^*/\beta)/\beta.
\]
Hence, the usage prices at the symmetric equilibrium satisfy

$$\hat{r}^* = -m$$

and

$$\hat{p}^* = -m/\beta.$$  \hfill (28)

Note that $\hat{r}^* \geq \beta c/(\beta + 1)$ or, equivalently, $\hat{p}^* + \hat{r}^* \geq c$ if and only if $m \leq -\beta c/(\beta + 1)$.

Define

$$\bar{m} = -\beta c \frac{1}{1 + \beta}.$$  \hfill (29)

**Proposition 5** Define

$$\hat{p}(m) = c + m \text{ and } \hat{r}(m) = \beta(c + m) \text{ for } m \geq \bar{m}$$

and

$$\hat{p}(m) = -m/\beta \text{ and } \hat{r}(m) = -m \text{ for } m \leq \bar{m}.$$  \hfill (30)

Then $(\hat{p}(m), \hat{r}(m))$ are the off-net prices in any symmetric equilibrium. In particular, for $m \geq \bar{m}$

$$\frac{\partial \pi_i}{\partial \hat{p}_i}(\hat{p}(m), \hat{r}(m)) = 0 \text{ and } \frac{\partial \pi_i}{\partial \hat{r}_i}(\hat{p}(m), \hat{r}(m)) \leq 0$$

and for $m \leq \bar{m}$

$$\frac{\partial \pi_i}{\partial \hat{r}_i}(\hat{p}(m), \hat{r}(m)) = 0 \text{ and } \frac{\partial \pi_i}{\partial \hat{p}_i}(\hat{p}(m), \hat{r}(m)) \leq 0.$$  \hfill (31)

**Proof.** This follows from the fact that for $m \geq \bar{m}$, $\hat{r}(m) \geq -m$ and that for $m \leq \bar{m}$, $\hat{p}(m) \geq c + m$.  \hfill \Box

Note that usage prices fall in termination mark-up for $m < \bar{m}$ and increase for $m > \bar{m}$ as illustrated in Fig. 3.

### 4.2 Fixed fee and market penetration

Let $\hat{R}(m) = (\hat{p}(m) + \hat{r}(m) - c)q(\hat{p}(m))$. We can rewrite profit as follows:

$$\pi_i = \alpha_i((1 - \alpha_i - \alpha_0)\hat{R}(m) + F_i - f).$$

Using

$$\partial \alpha_i / \partial F_i = -\alpha_i(1 - \alpha_i) / \mu$$
Figure 3: Off-net call and reception prices are minimal at \( m = \bar{m} \).

and

\[
\partial \alpha_0 / \partial F_i = \alpha_i \alpha_0 / \mu,
\]

we obtain the following first-order condition:

\[
0 = \frac{\partial \pi_i}{\partial F_i} = -\frac{\alpha_i (1 - \alpha_i)}{\mu} \left[ (1 - \alpha_i - \alpha_0) \hat{R}(m) + F_i - f \right] \\
+ \alpha_i \left( \left[ \frac{\alpha_i (1 - \alpha_i)}{\mu} - \frac{\alpha_i \alpha_0}{\mu} \right] \hat{R}(m) + 1 \right).
\]

In the symmetric equilibrium with fulfilled expectations \( \alpha_i = \bar{\beta} = \bar{\alpha} \) and \( \alpha_0 = 1 - n\bar{\alpha} \) so that we must have

\[
F^* = f + \frac{\mu}{1 - \bar{\alpha}} + (n - 1) \frac{\bar{\alpha} (2\bar{\alpha} - 1)}{1 - \bar{\alpha}} \hat{R}(m)
\]

(29)

We will denote the right-hand side of the above expression by \( F^{FOC}(\bar{\alpha}, m) \) and refer to it as the equilibrium curve. It describes a relation between the equilibrium fixed fee and the number of subscribers per firm in equilibrium. This fixed fee leads to equilibrium profit equal to

\[
\pi^* = \frac{\bar{\alpha}}{1 - \bar{\alpha}} \left( \mu + \bar{\alpha}^2 (n - 1) \hat{R}(m) \right).
\]

There is of course a second relation between equilibrium fixed fee and number of subscribers per firm. Namely, rational expectations in the Logit model require

\[
\bar{\alpha} = \frac{\exp(w^*/\mu)}{n \exp(w^*/\mu) + \exp(w_0/\mu)},
\]

(30)
where \( w^* \) denotes the expected surplus a subscriber gets from subscribing to one of the networks, that is

\[
    w^* = \bar{\alpha}(1 + \beta)(u(q(p^*)) - p^*q(p^*)) + (n - 1)\bar{\alpha}(1 + \beta)[u(q(\hat{p}^*)) - \hat{p}^*q(\hat{p}^*)] - F^*.
\]

Denoting \( v(p) = u(q(p)) - pq(p) \) equation (30) can be rewritten as

\[
    F^* = \bar{\alpha}(1 + \beta)\left[v(p^*) + (n - 1)v(\hat{p}(m))\right] - w_0 - \mu \log \left(\frac{\bar{\alpha}}{1 - n\bar{\alpha}}\right) \tag{31}
\]

We call the right-hand side of equation (31) the rational expectations curve \( F_{RE}(\bar{\alpha}, m) \).

The equilibrium fixed fee and number of subscribers per firm are found by solving equations (29) and (31). Note that at \( m = \bar{m} \), \( F_{FOC}(\bar{\alpha}, \bar{m}) = f + \mu/(1 - \bar{\alpha}) \) is increasing in \( \bar{\alpha} \) while the rational expectations curve is downward sloping for sufficiently high \( \mu \). Since the rational expectations curve has vertical asymptotes at \( \bar{\alpha} = 0 \) and at \( \bar{\alpha} = 1/n \), a solution exists and is necessarily unique.

**Lemma 6** For \( |m - \bar{m}| \) small enough and \( \mu > (1 + \beta)v(p^*)/4 \) the system of equations (29) and (31) has a unique solution.

**Proof.** A sufficient condition for the existence and uniqueness of a solution is that the rational expectation curve is downward sloping everywhere.

\[
    \frac{\partial F_{RE}}{\partial \bar{\alpha}}(\bar{\alpha}, \bar{m}) = (1 + \beta)n v(p^*) - \frac{\mu}{\bar{\alpha}(1 - n\bar{\alpha})} < 0.
\]

By continuity, the result holds for \( |m - \bar{m}| \) not too large. ■

### 4.3 Comparative statics

We now investigate how the equilibrium behaves in a neighborhood of \( m = \bar{m} \). We need to take into account that the usage prices are not differentiable at \( \bar{m} \) since the type of equilibrium switches at this critical value. We will handle this by considering right- and left derivatives separately.

We first establish that the number of subscribers and the equilibrium fixed fee is maximized at \( m = \bar{m} \). That is, both increasing and decreasing \( m \) away from \( \bar{m} \) reduces consumer surplus.

**Proposition 7**

1. Increasing \( m \) above \( \bar{m} \) reduces overall subscription and equilibrium fixed fees.
2. Decreasing \( m \) below \( \bar{m} \) reduces overall subscription and equilibrium fixed fees.

**Proof.** See the Appendix.

From Proposition (5) we know that usage prices increase as termination mark-up is moved away from \( \bar{m} \) (in either direction). For \( m > \bar{m} \) origination of off-net calls is priced at perceived marginal cost \( c + m \) while reception is charged above the cost of termination. In this case consumers come with termination rents. On the other hand, if \( m < \bar{m} \), reception is charged at perceived marginal cost \(-m\) while origination is charged above the perceived marginal cost. In this case consumers come with origination rents. In both cases, competition for consumers becomes fiercer and this leads firms to charge lower fixed fees. This means there is a waterbed effect at play. However, consumers are not fully compensated by lower fixed fees for the higher call and reception prices as is reflected by the lower overall subscription rates. This means the waterbed is partial and not full.

It is not obvious how equilibrium profit responds to a change in the termination mark-up away from \( \bar{m} \). Namely, such a change leads to higher termination or origination profits per subscriber, but it also leads to less subscribers. We will now analyze the effect of the termination mark-up on equilibrium profit and total welfare.

We first analyze how profits change along the rational expectation curve \( F^{RE}(\bar{\alpha}, \bar{m}) \) (that is, when termination mark-up is fixed at \( \bar{m} \)) as market penetration is varied. Note that since \( \hat{R}(\bar{m}) = 0 \), profits in this case are equal to \( \pi = \bar{\alpha}(F^{RE}(\bar{\alpha}, \bar{m}) - f) \), so that

\[
\frac{\partial \pi}{\partial \bar{\alpha}} = F^{RE}(\bar{\alpha}, \bar{m}) - f + \bar{\alpha} \frac{\partial F^{RE}}{\partial \bar{\alpha}}.
\]

Using that at \( m = \bar{m} \), \( F^{RE}(\bar{\alpha}, \bar{m}) = F^{FOC}(\bar{\alpha}, \bar{m}) = f + \mu/(1 - \bar{\alpha}) \), we obtain

\[
\frac{\partial \pi}{\partial \bar{\alpha}} = \bar{\alpha} n(1 + \beta)v(p^*)(1 - \bar{\alpha})(1 - n\bar{\alpha}) - (n - 1) \mu
\]

\[
(1 - \bar{\alpha})(1 - n\bar{\alpha}).
\]

The sign is ambiguous since it is negative for markets with high penetration (when \( \bar{\alpha} \approx 1/n \)) while it is positive for \( \bar{\alpha} \approx 0 \) and \( \mu < n(1 + \beta)v(p^*)(1 - \bar{\alpha})(1 - n\bar{\alpha})/(n - 1) \). If the sign is negative, it means that if the firms would act as a cartel, they would prefer to increase fixed fees and have less market penetration. As in Hurkens and López we refer to this case as one of effective competition (since competition between firms leads to lower fixed fees and higher market penetration). On the other hand, if the sign is positive it means that even colluding firms would prefer to increase market penetration and lower fixed fees. This is the case when externalities are very strong but are not well internalized under competition.

In order to analyze the effect of the termination mark-up on profit, let us denote \( \alpha(m) \) for the equilibrium number of subscribers per firm. Equilibrium profit per firm, as a function
of \( m \), can now be written as

\[
\Pi(m) = \alpha(m)(F^{RE}(\alpha(m), m) - f + (n - 1)\alpha(m)\check{R}(m)).
\]

Let

\[
\check{R}'_+(\bar{m}) = \lim_{m \uparrow \bar{m}} \frac{\check{R}(m) - \hat{r}(\bar{m})}{m - \bar{m}} = (1 + \beta)q(p^*).
\]

Similarly, let

\[
\check{R}'_-(\bar{m}) = \lim_{m \downarrow \bar{m}} \frac{\check{R}(m) - \hat{r}(\bar{m})}{m - \bar{m}} = -\frac{(1 + \beta)q(p^*)}{\beta}.
\]

Note that

\[
\frac{\partial F^{RE}}{\partial m} = -\alpha(\bar{m})(n - 1)\check{R}'_+(\bar{m})
\]

and

\[
\frac{\partial F^{RE}}{\partial m} = -\alpha(\bar{m})(n - 1)\check{R}'_-(\bar{m}).
\]

Since \( \check{R}(\bar{m}) = 0 \) we obtain

\[
\Pi'(\bar{m}) = \alpha'(\bar{m}) \left( F^{RE}(\alpha(\bar{m}), \bar{m}) - f + \alpha(\bar{m})\frac{\partial F^{RE}}{\partial \alpha} \right)
\]

(32)

The expression between brackets is the derivative with respect to \( \alpha \) of profits along the rational expectation curve. If profits increase along the rational expectation curve, then any change away from \( \bar{m} \) lowers profits. Hence, in this case firms strictly prefer to have \( m = \bar{m} \).

On the other hand, if profits decrease along the rational expectations curve, then firms prefer a termination mark-up below or above \( \bar{m} \).

Let us finally consider total surplus. Total surplus is equal to the sum of consumer surplus and industry profit, that is,

\[
TS = CS + n\pi.
\]

Since, \( dCS/dm = (\partial \bar{\alpha}/\partial m)(n\mu/(1 - n\bar{\alpha})) \), we have

\[
\frac{dT S}{d m} = \left( \frac{\partial \bar{\alpha}}{\partial m} \right) \left( \frac{n\mu}{1 - n\bar{\alpha}} + n \left( F^{RE} - f + \bar{\alpha}\frac{\partial F^{RE}}{\partial \bar{\alpha}} \right) \right)
\]

\[
= n \left( \frac{\partial \bar{\alpha}}{\partial m} \right) \left( n\bar{\alpha}(1 + \beta)v(p^*) + \frac{\mu}{1 - \bar{\alpha}} \right).
\]

Since the second factor is positive, total surplus is maximized when consumer surplus is maximized, that is, when \( m = \bar{m} \).

**Proposition 8** Consumer and total surplus are maximized at \( m = \bar{m} \). Industry profit is
maximized at $m = \bar{m}$ if and only if network externalities are very strong. Otherwise firms prefer either a lower or a higher termination mark-up.

Note that if $m > \bar{m}$, then we can define $\tilde{m}$ as $\tilde{m} = (1 + \beta)\bar{m} - \beta m$. Then $\tilde{m} < \bar{m}$ and $\dot{p}(\tilde{m}) = \dot{p}(m)$, and also $\dot{R}(\tilde{m}) = \dot{R}(m)$. As long as $\tilde{m} \geq -c_T$, firms can obtain the maximum profit both with some $m > \tilde{m}$ and with the corresponding $\tilde{m} < \bar{m}$. This is illustrated by Fig. 4.

![Figure 4: Profits are maximal with $m \neq \bar{m}$ when competition is effective.](image)

Figure 4: Profits are maximal with $m \neq \bar{m}$ when competition is effective.

![Figure 5: Consumer surplus is always maximized at $m = \bar{m}$.](image)

Figure 5: Consumer surplus is always maximized at $m = \bar{m}$. 34
5 Concluding remarks

In this paper we revisit the analysis of the effect of termination charges on competition and welfare when receivers obtain utility from incoming calls and network operators can charge call reception. Compared to earlier literature on this topic we assume that consumers form expectations about network sizes in a passive, but ex-post rational way. In addition, we extend the traditional duopoly model to the case of competition among multiple networks, and allow both callers and receivers to hang up. We solve the model in the absence of noise and consider the case of elastic subscription demand. Let us first recap the main results.

First, we consider the case in which there is full participation. Both in the absence and presence of network-based discrimination there exist multiple symmetric equilibria. We have showed that in the absence of network-based discrimination, at any symmetric equilibrium, profit is neutral to the access charge and equals the profit that firms would obtain under unit demand. When we allow for off-net/on-net call charge differentials we show that the number of competitors is important to determine whether firms have incentives to create connectivity breakdown by setting a too high off-net reception (respectively, calling) price when at equilibrium callers (respectively, receivers) determine the call volume. We showed that if the number of firms is relatively high, then there will not be incentives to provoke connectivity breakdown. We further showed that equilibrium profit is no longer neutral to the termination rate, and varies as a function of the particular equilibrium that is considered.

Second, to deal with multiplicity of equilibria, we discussed three possible equilibrium selection hypotheses:

(i) *When firms use the CPP regime as a selection device.* In the absence of network-based discrimination, equilibrium profit is neutral to the termination rate, and the socially optimal termination mark-up is below cost and increases with the number of firms. In the presence of network-based discrimination, firms prefer termination mark-up that yields off-net price equal to monopoly price. This implies that it will be negative for the case of two firms and strong call externality, but strictly positive for industries with at least three firms or weaker call externality.

(ii) *Noisy receiver utility.* In the absence of network-based discrimination, this equilibrium selection criterion yields off-net cost prices. It follows that profit is not affected by the level of the termination rate and that the socially optimal termination mark-up is below cost (there is scope for bill-and-keep to be socially optimal). When network-based price discrimination is allowed, we have showed that firms prefer a negative termination mark-up if and only if the call externality is sufficiently strong in relation to the elasticity of call demand.
(iii) Equilibrium that yields highest profit. We have showed that when there is network-based price discrimination and more than two firms in the industry, this equilibrium is the one in which callers and receivers would hang up at the same time; this maximizes the volume of calls and the sum of usage prices.

For all equilibrium selection hypotheses we find that the socially optimal termination charge is negative, but that it is even lower when firms use CPP. This suggests that disputes between regulators and network operators are more polarized in countries with CPP regimes.

Third, we examined the case of elastic subscription demand with network-based discrimination. We were able to characterize equilibria by focusing on a particular type of equilibrium in which a marginal increase in call price does not affect the surplus of subscribing to the rival networks. We derived an existence result for retail price equilibrium. We also showed that the termination charge that maximizes consumer and total surplus is below cost but in general is positive, not zero. This implies that setting a positive termination mark-up does not stimulate market penetration. Indeed, any departure (in either direction) from the socially optimal termination mark-up reduces overall subscription and equilibrium fixed fees. Finally, we showed that industry profit is maximized at the socially optimal termination mark-up if and only if network externalities are very strong. Otherwise firms prefer either a lower or a higher termination mark-up.

Appendix

Proof. Lemma 2.

Suppose a network considers to raise the reception charge for off-net calls above $\beta \hat{p}^*$. Such a deviation would make the receivers of this network determine the volume of calls received from subscribers from rival networks. All firms $j \neq i$ set $\hat{p}_j = \hat{p}^*$, $\hat{r}_j = \hat{r}^*$, and $F_j = F^*$ where $\hat{p}^*$ and $\hat{r}^*$ satisfy (19) and $F^*$ satisfies (20) while firm $i$ sets $\hat{p}_i = \hat{p}^*$, $\hat{r}_i > \beta \hat{p}^*$, and $F_i$.

The profit of firm $i$ is then equal to

$$
\pi_i = \alpha_i((1 - \alpha_i)(\hat{p}^* - c - m)q(\hat{p}^*) + (1 - \alpha_i)(\hat{r}_i + m)q(\hat{r}_i/\beta) + F_i - f).
$$

As before, when considering an alternative reception charge $\hat{r}_i$ one can keep market share constant at $1/n$ by adjusting $F_i$ accordingly. That is,

$$
F_i = \frac{n-1}{n} \left[ (\beta u(q(\hat{r}_i/\beta)) - \hat{r}_i q(\hat{r}_i/\beta)) - (\beta u(q(\hat{p}^*)) - \hat{r}_i q(\hat{p}^*)) \right] \\
+ \frac{1}{n} \left[ (u(q(\hat{p}^*) - \hat{p}^* q(\hat{p}^*)) - (u(q(\hat{r}_i/\beta)) - \hat{p}^* q(\hat{r}_i/\beta)) \right] + F^*
$$
Observe that
\[ \frac{\partial F_i}{\partial \hat{r}_i} = \frac{n - 1}{n}[-q(\hat{r}_i/\beta)] - \frac{1}{n\beta^2}[(\hat{r}_i - \beta \hat{p}^*)q'(\hat{r}_i/\beta)]. \]

Keeping market share \( \alpha_i \) constant at \( 1/n \), the first-order derivative of profit w.r.t. \( \hat{r}_i \) is
\[
\frac{\partial \pi_i}{\partial \hat{r}_i} = \frac{1}{n} \left[ \frac{n - 1}{n} \left[ q(\hat{r}_i/\beta) + (\hat{r}_i + m)q'(\hat{r}_i/\beta) - q(\hat{r}_i/\beta) \right] - \frac{1}{n\beta^2} (\hat{r}_i - \beta \hat{p}^*)q'(\hat{r}_i/\beta) \right]
\]
\[= \frac{q'(\hat{r}_i/\beta)}{\beta^2 n^2} \left[ \beta(n - 1)(\hat{r}_i + m) - \hat{r}_i + \beta \hat{p}^* \right].\]

Note that if \((n - 1)\beta - 1 < 0\), the profit function is U-shaped while if \((n - 1)\beta - 1 > 0\), the profit function is inversely U-shaped. Moreover, at \( \hat{r}_i = \beta \hat{p}^* \)
\[\frac{\partial \pi_i}{\partial \hat{r}_i} > 0 \text{ if and only if } \beta \hat{p}^* + m < 0.\]

Hence, if \( \beta \hat{p}^* + m < 0 \) firm \( i \) will certainly want to deviate since even a marginal deviation above \( \beta \hat{p}^* \) would be profitable. On the other hand, if \( \beta \hat{p}^* + m > 0 \) marginal deviations are not profitable. If moreover, \((n - 1)\beta - 1 > 0\), then there is no profitable deviation at all (in which market shares are kept constant). Finally, if \( \beta \hat{p}^* + m > 0 \) and \((n - 1)\beta - 1 < 0\) deviating to \( \hat{r}_i = \infty \) may be profitable.

**Proof. Lemma 3**

We will now check whether a firm \( i \) may have an incentive to raise the off-net call price \( \hat{p}_i \) above \( \hat{r}^*/\beta \). Such a deviation makes the callers of this network determine the volume of off-net calls. Profit of firm \( i \) is then equal to
\[ \pi_i = \alpha_i(1 - \alpha_i)(\hat{p}_i - c - m)q(\hat{p}_i) + (1 - \alpha_i)(\hat{r}^* + m)q(\hat{r}^*/\beta) + F_i - f. \]

As before, when considering an alternative reception price, one can keep market share constant by adjusting \( F_i \) accordingly. That is
\[ F_i = \frac{n - 1}{n} \left[ (u(q(\hat{p}_i)) - \hat{p}_i q(\hat{p}_i)) - (u(q(\hat{r}^*/\beta)) - \hat{p}^* q(\hat{r}^*/\beta)) \right]
\]
\[+ \frac{1}{n} \left[ ((\beta u(q(\hat{r}^*/\beta)) - \hat{r}^* q(\hat{r}^*/\beta)) - (\beta u(q(\hat{p}_i)) - \hat{r}^* q(\hat{p}_i)) \right] + F^*.\]

Observe that
\[ \frac{\partial F_i}{\partial \hat{p}_i} = \frac{n - 1}{n}[-q(\hat{p}_i)] - \frac{1}{n}[(\beta \hat{p}_i - \hat{r}^*) q'(\hat{p}_i)]. \]
\[ 0 = \partial \pi / \partial \hat{p}_i = \alpha \left[ \frac{n-1}{n} [g(\hat{p}_i) + (\hat{p}_i - c - m)q'(\hat{p}_i) - q(\hat{p}_i)] - \frac{1}{n} (\beta \hat{p}_i - \hat{r}^*) q'(\hat{p}_i) \right] \]
\[ = \frac{q'(\hat{p}_i)}{n^2} [(n - 1 - \beta) \hat{p}_i - (n - 1)(c + m) + \hat{r}^*] \]

so that

\[ (n - 1 - \beta) \hat{p}_i - (n - 1)(c + m) + \hat{r}^* = 0. \]

Note that the second-order derivative of profits, evaluated at the solution of the first-order condition, reads
\[ \frac{\partial^2 \pi}{\partial \hat{p}_i^2} = \frac{q'(\hat{p}_i)}{n^2} (n - 1 - \beta) < 0 \]
for all \( \beta < 1 \) and \( n \geq 2 \). A profitable (marginal) deviation above \( \hat{r}^*/\beta \) thus exists whenever \( \partial \pi / \partial \hat{p}_i > 0 \), when evaluated at \( \hat{p}_i = \hat{r}^*/\beta \). So a necessary condition for the equilibrium candidate to be an equilibrium is that \( \hat{r} \geq \beta(c + m) \).

**Proof. Proposition 7**

1. Let
\[ \frac{\partial F^RE}{\partial m} = \lim_{m \downarrow \bar{m}} \frac{F^RE(\bar{\alpha}, m) - F^RE(\bar{\alpha}, \bar{m})}{m - \bar{m}}. \]

Since \( v'(p) = -q(p), \hat{r}'(m) = 1 \) (for \( m \geq \bar{m} \)) and \( \hat{p}(\bar{m}) = p^* \), it follows that
\[ \frac{\partial F^RE}{\partial m} = -\bar{\alpha}(1 + \beta)(n - 1)q(p^*). \]

Let
\[ \frac{\partial F^FOC}{\partial m} = \lim_{m \downarrow \bar{m}} \frac{F^FOC(\bar{\alpha}, m) - F^FOC(\bar{\alpha}, \bar{m})}{m - \bar{m}}. \]

It follows that
\[ \frac{\partial F^FOC}{\partial m} = -\bar{\alpha}(1 + \beta)(n - 1)q(p^*) \frac{1 - 2\bar{\alpha}}{1 - \bar{\alpha}}. \]

Hence
\[ \frac{\partial F^RE}{\partial m} < \frac{\partial F^FOC}{\partial m} < 0. \]

This means that an increase of termination mark-up \( m \) shifts down the rational expectations curve by more than the equilibrium curve. The intersection point thus shifts to the south-west, lowering both the number of subscribers and the equilibrium fixed fee.

2. Let
\[ \frac{\partial F^RE}{\partial m} = \lim_{m \downarrow \bar{m}} \frac{F^RE(\bar{\alpha}, m) - F^RE(\bar{\alpha}, \bar{m})}{m - \bar{m}}. \]
Since \( v'(p) = -q(p) \), \( \hat{p}'(m) = -1/\beta \) (for \( m \leq \bar{m} \)) and \( \hat{p} (\bar{m}) = p^* \), it follows that

\[
\frac{\partial F_{RE}}{\partial m} = \frac{\alpha (1 + \beta)(n - 1)q(p^*)}{\beta}.
\]

Let

\[
\frac{\partial F_{FOC}^-}{\partial m} = \lim_{m \uparrow \bar{m}} \frac{F_{FOC} (\bar{\alpha}, m) - F_{FOC} (\bar{\alpha}, \bar{m})}{m - \bar{m}}.
\]

It follows that

\[
\frac{\partial F_{FOC}^-}{\partial m} = \frac{\bar{\alpha} (1 + \beta)(n - 1)q(p^*)}{\beta} \frac{1 - 2\bar{\alpha}}{1 - \bar{\alpha}}.
\]

Hence

\[
\frac{\partial F_{RE}}{\partial m} > \frac{\partial F_{FOC}^-}{\partial m} > 0.
\]

This means that a decrease of termination mark-up \( m \) shifts down the rational expectations curve by more than the equilibrium curve. The intersection point thus shifts to the south-west, lowering both the number of subscribers and the equilibrium fixed fee. ■

References


